

内潮的一种分层三维数值模式*

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在中国近海的不少海域曾经发现相当强的内潮现象,例如赵俊生(1992)报道了渤海海峡的内潮现象,Yamashiro(1988)在东海陆坡处亦发现内潮的存在。特别是 Amoco 石油公司在南海陆坡处进行了一年多的海流测量,测得最大的全日潮流振幅可超过 60 cm/s,这里正压潮流小于 10cm/s,故观测到的潮流主要与内潮有关。迄今关于内潮的数值模式大多将表面潮作为已知输入条件,从而使模式的应用受到较多限制(Jiang and Fang, 1992)。Heaps(1983)曾发展了一种自由海面多层模式,但他们忽略了非线性平流项,这无疑也会损失许多有用信息。本文将叙述一种自由海面的多层模式,它能够同时模拟表面潮和内潮,并包含了非线性平流项。

一、基本方程

我们把海洋分成 K 层,用 $k=1,2,\dots,K$ 记之。每层具有相同的海水密度 ρ_k ,即 ρ_k 与

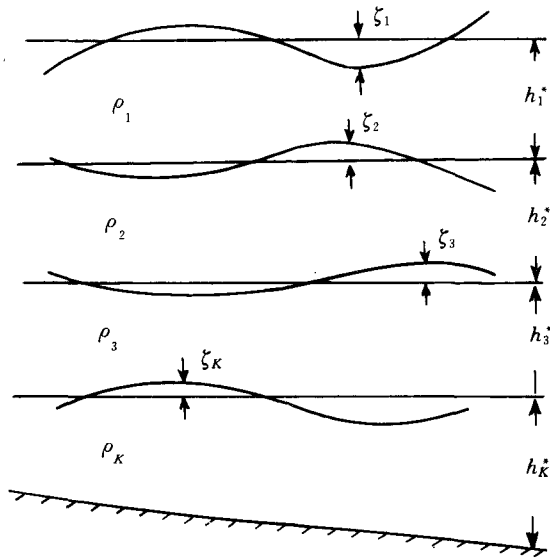


图 1 垂向分层

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层号 k 有关,与水平位置 (x, y) 无关。各层静止时的厚度除底层外也只与 k 有关,与 (x, y) 无关,记为 $h_1^*, h_2^*, \dots$, 各层上界面的扰动高度记为 ζ_k (如图 1)。以 u_k 和 v_k 记各层水平流速的 x 和 y 方向分量; τ_{kx} 和 τ_{ky} 记第 k 层与第 $k+1$ 层之间摩擦力的 x 和 y 方向分量,特别地, τ_{sx}, τ_{sy} 和 τ_{Kx}, τ_{Ky} 分别为海面风应力和海底处摩擦力; h 为未扰动总水深, f 和 g 分别为 Coriolis 参量和重力加速度, P_a 为海面大气压, A 为水平湍粘系数,则基本方程可写为:

$$\begin{cases} \frac{\partial u_1}{\partial t} = -L_1(u_1) + fv_1 - g \frac{\partial \zeta_1}{\partial x} - \frac{1}{\rho_1} \frac{\partial p_a}{\partial x} + \frac{\tau_{sx} - \tau_{1x}}{h_1 + \zeta_1 - \zeta_2} + A\Delta u_1 \\ \frac{\partial v_1}{\partial t} = -L_1(v_1) - fu_1 - g \frac{\partial \zeta_1}{\partial y} - \frac{1}{\rho_1} \frac{\partial p_a}{\partial y} + \frac{\tau_{sy} - \tau_{1y}}{h_1 + \zeta_1 - \zeta_2} + A\Delta v_1 \\ \frac{\partial \zeta_1}{\partial t} = \frac{\partial \zeta_2}{\partial t} - \frac{\partial[(h_1 + \zeta_1 - \zeta_2)u_1]}{\partial x} - \frac{\partial[(h_1 + \zeta_1 - \zeta_2)v_1]}{\partial y} \end{cases} \quad (1.1)$$

$$\begin{cases} \frac{\partial u_2}{\partial t} = -L_2(u_2) + fv_2 - g \frac{\rho_1}{\rho_2} \frac{\partial \zeta_1}{\partial x} - g \frac{\rho_2 - \rho_1}{\rho_2} \frac{\partial \zeta_2}{\partial x} - \frac{1}{\rho_2} \frac{\partial p_a}{\partial x} + \frac{\tau_{1x} - \tau_{2x}}{h_2 + \zeta_2 - \zeta_3} + A\Delta u_2 \\ \frac{\partial v_2}{\partial t} = -L_2(v_2) - fu_2 - g \frac{\rho_1}{\rho_2} \frac{\partial \zeta_1}{\partial y} - g \frac{\rho_2 - \rho_1}{\rho_2} \frac{\partial \zeta_2}{\partial y} - \frac{1}{\rho_2} \frac{\partial p_a}{\partial y} + \frac{\tau_{1y} - \tau_{2y}}{h_2 + \zeta_2 - \zeta_3} + A\Delta v_2 \\ \frac{\partial \zeta_2}{\partial t} = \frac{\partial \zeta_3}{\partial t} - \frac{\partial[(h_2 + \zeta_2 - \zeta_3)u_2]}{\partial x} - \frac{\partial[(h_2 + \zeta_2 - \zeta_3)v_2]}{\partial y} \end{cases} \quad (1.2)$$

$$\begin{cases} \frac{\partial u_K}{\partial t} = -L_K(u_K) + fv_K - g \frac{\rho_1}{\rho_K} \frac{\partial \zeta_1}{\partial x} - g \frac{\rho_2 - \rho_1}{\rho_K} \frac{\partial \zeta_2}{\partial x} - \dots \\ \quad - g \frac{\rho_{K-1} - \rho_{K-2}}{\rho_K} \frac{\partial \zeta_{K-1}}{\partial x} - g \frac{\rho_K - \rho_{K-1}}{\rho_K} \frac{\partial \zeta_K}{\partial x} \\ \quad - \frac{1}{\rho_K} \frac{\partial p_a}{\partial x} + \frac{\tau_{K-1x} - \tau_{Kx}}{h_K + \zeta_K} + A\Delta u_K \\ \frac{\partial v_K}{\partial t} = -L_K(v_K) - fu_K - g \frac{\rho_1}{\rho_K} \frac{\partial \zeta_1}{\partial y} - g \frac{\rho_2 - \rho_1}{\rho_K} \frac{\partial \zeta_2}{\partial y} - \dots \\ \quad - g \frac{\rho_{K-1} - \rho_{K-2}}{\rho_K} \frac{\partial \zeta_{K-1}}{\partial y} - g \frac{\rho_K - \rho_{K-1}}{\rho_K} \frac{\partial \zeta_K}{\partial y} \\ \quad - \frac{1}{\rho_K} \frac{\partial p_a}{\partial y} + \frac{\tau_{K-1y} - \tau_{Ky}}{h_K + \zeta_K} + A\Delta v_K \\ \frac{\partial \zeta_K}{\partial t} = - \frac{\partial[(h_K + \zeta_K)u_K]}{\partial x} - \frac{\partial[(h_K + \zeta_K)v_K]}{\partial y} \end{cases} \quad (1.3)$$

其中,

$$L_k(a) = \frac{\partial}{\partial x}(u_k a) + \frac{\partial}{\partial y}(v_k a), \quad \Delta a = \frac{\partial^2 a}{\partial x^2} + \frac{\partial^2 a}{\partial y^2} \quad (1.4)$$

$$\begin{cases} \frac{\partial u}{\partial t} = -L(u) + fv - g \frac{\partial \zeta_1}{\partial x} - \frac{1}{\rho} \frac{\partial p_a}{\partial x} + \frac{\tau_{sx} - \tau_{Kx}}{h + \zeta_1} + A \Delta u \\ \frac{\partial v}{\partial t} = -L(v) + fv - g \frac{\partial \zeta_1}{\partial y} - \frac{1}{\rho} \frac{\partial p_a}{\partial y} + \frac{\tau_{sy} - \tau_{Ky}}{h + \zeta_1} + A \Delta v \\ \frac{\partial \zeta}{\partial t} = - \frac{\partial [(h + \zeta_1)u]}{\partial x} - \frac{\partial [(h + \zeta_1)v]}{\partial y} \end{cases} \quad (1.5)$$
$$L(a) = \frac{1}{h + \zeta_1} \sum_{k=1}^K [H_k L_k(a_k)] \quad (1.6)$$
$$\zeta \text{ 点: } \zeta_1(i, j), \zeta_2(i, j), \zeta_3(i, j), \tilde{p}(i, j)$$

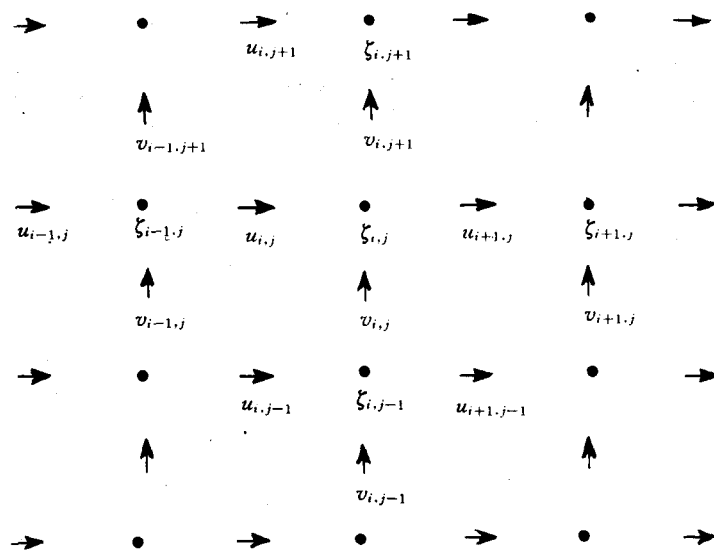


图2 水平网格点配置图

u 点: $h'(i, j), K'(i, j), h'_1(i, j), h'_2(i, j), h'_3(i, j), \beta'(i, j), g'_2(i, j), g'_3(i, j), \tau_{xx}(i, j),$
 $u(i, j), u_1(i, j), u_2(i, j), u_3(i, j), Q_x(i, j)$

v 点: $h''(i, j), K''(i, j), h''_1(i, j), h''_2(i, j), h''_3(i, j), \beta''(i, j), g''_2(i, j), g''_3(i, j), \tau_{yy}(i, j),$
 $v(i, j), v_1(i, j), v_2(i, j), v_3(i, j), Q_y(i, j)$

此外,在模式中包含有下列参数:

$\rho_1, \rho_2, \rho_3, h_1^*, h_2^*, g, f, A, C, \Delta x, \Delta y, \Delta t, T_x (= \Delta t / (2\Delta x)), T_y (= \Delta t / (2\Delta y)), C_r (= f\Delta t / 2), v_2, v_3$

其中 v_2 和 v_3 为第一、二层之间及第二、三层之间的垂直湍粘系数。上列变量和参数的计算方法如下:

1. $\tilde{p} \equiv p_a / \rho_1$, 若 p_a 单位取 mb (即 hPa), 则 $\tilde{p} = p_a \div 102.5$, 单位: m 。
2. 如水深 $h_{i,j}$ 在水平网格空格处 (即 $v_{i-1,j}$ 和 $v_{i,j}$ 之间, 亦即 $u_{i,j-1}$ 和 $u_{i,j}$ 之间) 给出, 则可事先算出 $h'_{i,j}$ 和 $h''_{i,j}$, 算完后 $h_{i,j}$ 即可不必保留。计算公式为

$$h'_{i,j} = (h_{i,j} + h_{i,j+1}) / 2, \quad h''_{i,j} = (h_{i,j} + h_{i+1,j}) / 2$$

3. K', h'_1, h'_2, h'_3 的确定方法如下:

若 $h''_{i,j} \leq h_1^*$, 则 $K'_{i,j} = 1, h'_{1i,j} = h'_{i,j}$

若 $h_1^* < h'_{i,j} \leq h_1^* + h_2^*$, 则 $K'_{i,j} = 2, h'_{1i,j} = h_1^*, h'_{2i,j} = h'_{i,j} - h_1^*$

若 $h_1^* + h_2^* < h'_{i,j}$, 则 $K'_{i,j} = 3, h'_{1i,j} = h_1^*, h'_{2i,j} = h_2^*, h'_{3i,j} = h'_{i,j} - h_1^* - h_2^*$

K'', h''_1, h''_2, h''_3 确定方法一样。

4. $g'_{2i,j} = g[h'_{2i,j}(\rho_2 - \rho_1)/\rho_2 + h'_{3i,j}(\rho_2 - \rho_1)/\rho_3]/h'_{i,j}$
 $g'_{3i,j} = g[h'_{3i,j}(\rho_3 - \rho_1)/\rho_3]/h'_{i,j}$
 $\beta'_{i,j} = [h'_{1i,j} + h'_{2i,j}(\rho_1/\rho_2) + h'_{3i,j}(\rho_1/\rho_3)]/h'_{i,j}$
 g''_2, g''_3, β'' 算法一样, 仅将 h', h'_k 换成 h'', h''_k 。

三、有限差分方程

设 $n\Delta t$ 时刻 ζ_1, u, v 已知, $n\Delta t$ 时刻 $u_k, v_k (k=1, 2, 3)$ 已知, $(n+1/2)\Delta t$ 时刻 $\zeta_k (k=2, 3)$ 已知, $(n+1/2)\Delta t$ 时刻 $\tilde{p}, \tau_{sx}, \tau_{sy}$ 已算出, 则下一个 Δt 时刻内的计算过程如下:

1. ζ_1, u, v 的计算

ζ_1, u, v 为与表面潮有关的量, 我们采用 Leendertse (1967) 的交替方向隐格式, 具体计算步骤如下:

(1) 第一个半步

(a) ζ_1, u , 用隐式计算, 方程组为

$$\begin{cases} A'_{i,j} \zeta_{1i-1,j}^{n+1/2} + B'_{i,j} u_{i,j}^{n+1/2} + C'_{i,j} \zeta_{1i,j}^{n+1/2} = F'_{i,j} \\ A_{i,j} u_{i,j}^{n+1/2} + B_{i,j} \zeta_{1i,j}^{n+1/2} + C_{i,j} u_{i+1,j}^{n+1/2} = F_{i,j} \end{cases} \quad (3.1)$$

其中

$$\begin{cases} A'_{i,j} = -T_x g \beta'_{i,j} \\ B'_{i,j} = 1 \\ C'_{i,j} = -A'_{i,j} \\ F'_{i,j} = u_{i,j}^n + (\Delta t/2) \{ f \bar{v}_{i,j}^n - Q_{xi,j} + A[(u_{i+1,j}^n + u_{i+1,j}^n - 2u_{i,j}^n)/\Delta x^2 \\ + (u_{i,j-1}^n + u_{i,j+1}^n - 2u_{i,j}^n)/\Delta y^2] \} \end{cases} \quad (3.2)$$

$$\begin{cases} A_{i,j} = -T_x h'_{i,j} \\ B_{i,j} = 1 \\ C_{i,j} = T_x h'_{i+1,j} \\ F_{i,j} = \zeta_{1i,j}^n - T_y (h''_{i,j+1} v_{i,j+1}^n - h''_{i,j} v_{i,j}^n) \end{cases} \quad (3.3)$$

式(3.2)中

$$\begin{aligned} Q_{xi,j} = & [g'_{2i,j}(\zeta_{2i,j}^{n+1/2} - \zeta_{2i-1,j}^{n+1/2}) + g'_{3i,j}(\zeta_{3i,j}^{n+1/2} - \zeta_{3i-1,j}^{n+1/2}) \\ & + \beta'_{i,j}(\tilde{p})_{i,j}^{n+1/2} - \tilde{p}_{i-1,j}^{n+1/2}]/\Delta x \\ & - \{ \tau_{sx,i,j}^{n+1/2} - C[(u_{K'i,j}^n)^2 + (\bar{v}_{K'i,j}^n)^2]^{1/2} u_{K'i,j}^n \} / h'_{i,j} \end{aligned} \quad (3.4)$$

(b) v 用显式计算, 公式为

$$\begin{aligned} v_{i,j}^{n+1/2} = & v_{i,j}^n - (\Delta t/2) \{ Q_{yi,j} + g\beta''_{i,j}(\zeta_{1i,j}^n - \zeta_{1i,j-1}^n)/\Delta y \\ & + A[(v_{i-1,j}^n + v_{i+1,j}^n - 2v_{i,j}^n)/\Delta x^2 \\ & + (v_{i,j-1}^n + v_{i,j+1}^n - 2v_{i,j}^n)/\Delta y^2] \} \end{aligned} \quad (3.5)$$

式中

$$\begin{aligned} Q_{yi,j} = & f\bar{u}_{i,j}^{n+1/2} + [g''_{2i,j}(\zeta_{2i,j}^{n+1/2} - \zeta_{2i,j-1}^{n+1/2}) + g''_{3i,j}(\zeta_{3i,j}^{n+1/2} - \zeta_{3i,j-1}^{n+1/2}) + \beta''_{i,j}(\tilde{p}_{i,j}^{n+1/2} - \tilde{p}_{i,j-1}^{n+1/2})]/\Delta y \\ & - \{ \tau_{yy,i,j}^{n+1/2} - C[(\bar{u}_{K^y,i,j}^n)^2 + (v_{K^y,i,j}^n)^2]^{1/2} v_{K^y,i,j}^n \} / h''_{i,j} \end{aligned} \quad (3.6)$$

(2) 第二半步

(a) ζ_1, v 用隐式计算, 方程组为

$$\begin{cases} A'_{i,j}\zeta_{i,j-1}^{n+1} + B'_{i,j}v_{i,j}^{n+1} + C'_{i,j}\zeta_{i,j}^{n+1} = F'_{i,j} \\ A_{i,j}v_{i,j}^{n+1} + B_{i,j}\zeta_{i,j}^{n+1} + C_{i,j}v_{i,j+1}^{n+1} = F_{i,j} \end{cases} \quad (3.7)$$

其中

$$\begin{cases} A'_{i,j} = -T_y g\beta''_{i,j} \\ B'_{i,j} = 1 \\ C'_{i,j} = -A'_{i,j} \\ F'_{i,j} = v_{i,j}^{n+1/2} - (\Delta t/2) \{ Q_{yi,j} - A[(v_{i-1,j}^{n+1/2} + v_{i+1,j}^{n+1/2} - 2v_{i,j}^{n+1/2})/\Delta x^2 \\ + (v_{i,j-1}^{n+1/2} + v_{i,j+1}^{n+1/2} - 2v_{i,j}^{n+1/2})/\Delta y^2] \} \end{cases} \quad (3.8)$$

$$\begin{cases} A_{i,j} = -T_y h''_{i,j} \\ B_{i,j} = 1 \\ C_{i,j} = T_y h''_{i,j+1} \\ F_{i,j} = \zeta_{1i,j}^{n+1/2} - T_x (h'_{i+1,j} u_{i+1,j}^{n+1/2} - h'_{i,j} u_{i,j}^{n+1/2}) \end{cases} \quad (3.9)$$

式(3.8)中 $Q_{yi,j}$ 见式(3.6)。

(b) u 用显式计算, 公式为

$$\begin{aligned} u_{i,j}^{n+1} = & u_{i,j}^{n+1/2} + (\Delta t/2) \{ f\bar{v}_{i,j}^{n+1} - g\beta'_{i,j}(\zeta_{1i,j}^{n+1/2} - \zeta_{1i-1,j}^{n+1/2}) - Q_{xi,j} \\ & + A[(u_{i-1,j}^{n+1/2} + u_{i+1,j}^{n+1/2} - 2u_{i,j}^{n+1/2})/\Delta x^2 + (u_{i,j-1}^{n+1/2} + u_{i,j+1}^{n+1/2} - 2u_{i,j}^{n+1/2})/\Delta y^2] \} \end{aligned} \quad (3.10)$$

上面各式中 $\bar{u}, \bar{v}$ 定义如下:

$$\bar{u}_{i,j} = (u_{i,j} + u_{i+1,j} + u_{i+1,j-1} + u_{i,j-1})/4, \bar{v}_{i,j} = (v_{i,j} + v_{i-1,j} + v_{i-1,j+1} + v_{i,j+1})/4 \quad (3.11)$$

2. $u_k, v_k (k=1, 2, 3), \zeta_k (k=2, 3)$ 的计算

(1) u_k, v_k 的计算

(a) u_k 的计算

若第 (i, j) 个 u 点的 $K'_{i,j} = 1$, 可直接取

$$u_{1i,j}^{n+1} = u_{i,j}^{n+1} \quad (3.12)$$

若 $K'_{i,j} = 2$, 则

$$\begin{cases} \hat{u}_{1i,j}^{n+1} = (B_2 F_1 - C_1 F_2) / (1 - A_2 - C_1) \\ \hat{u}_{2i,j}^{n+1} = (B_2 F_2 - A_2 F_1) / (1 - A_2 - C_1) \end{cases} \quad (3.13)$$

式中

$$\begin{cases} C_1 = -2v_2 \Delta t / [h'_{1i,j} (h'_{1i,j} + h'_{2i,j})] \\ B_1 = 1 - C_1 \\ A_2 = C_1 h'_{1i,j} / h'_{2i,j} \\ B_2 = 1 - A_2 \\ F_1 = u_{1i,j}^n + \Delta t \{ f \bar{v}_{1i,j}^n - G_1 + \tau_{xx}^{n+1/2} / h'_{1i,j} + A [(u_{1i-1,j}^n + u_{1i+1,j}^n - 2u_{1i,j}^n) / \Delta x^2 \\ + (u_{1i,j-1}^n + u_{1i,j+1}^n - 2u_{1i,j}^n) / \Delta y^2] \} \\ F_2 = u_{2i,j}^n + \Delta t \{ f \bar{v}_{2i,j}^n - G_2 - C [(u_{2i,j}^n)^2 + (\bar{v}_{2i,j}^n)^2]^{1/2} u_{2i,j}^n / h'_{2i,j} \\ + A [(u_{2i-1,j}^n + u_{2i+1,j}^n - 2u_{2i,j}^n) / \Delta x^2 + (u_{2i,j-1}^n + u_{2i,j+1}^n - 2u_{2i,j}^n) / \Delta y^2] \} \end{cases} \quad (3.14)$$

其中

$$\begin{cases} G_1 = [g(\zeta_{1i,j}^{n+1/2} - \zeta_{1i-1,j}^{n+1/2}) + (\tilde{p}_{i,j}^{n+1/2} - \tilde{p}_{i-1,j}^{n+1/2})] / \Delta x \\ G_2 = (\rho_1 / \rho_2) G_1 + g(1 - \rho_1 / \rho_2) (\zeta_{2i,j}^{n+1/2} - \zeta_{2i-1,j}^{n+1/2}) / \Delta x \end{cases} \quad (3.15)$$

最后取

$$u_{1i,j}^{n+1} = \hat{u}_{1i,j}^{n+1} + \Delta u, \quad u_{2i,j}^{n+1} = \hat{u}_{2i,j}^{n+1} + \Delta u \quad (3.16)$$

其中

$$\Delta u = u_{i,j}^{n+1} - (\hat{u}_{1i,j}^{n+1} h'_{1i,j} + \hat{u}_{2i,j}^{n+1} h'_{2i,j}) / h'_{i,j} \quad (3.17)$$

应注意的是,在计算 u_k, v_k 时用到 $\zeta_1^{n+1/2}$, 故对 $\zeta_{1(i,j)}$ 需两个数组。在计算出 $u^{n+1/2}$ 和 $\zeta_1^{n+1/2}$ 后, ζ_1 仍要保留, 用于计算 $v^{n+1/2}$ [见式(3.5)]; 在计算出 $v^{n+1/2}$ 和 ζ_1^{n+1} 后 $\zeta_1^{n+1/2}$ 可不要, 但 ζ_1^{n+1} 仍要保留, 用于计算 u_k, v_k 。

若 $K'_{i,j}=3$, 则需解下列三对角系数矩阵的方程组:

$$\begin{cases} B_1 \hat{u}_{1i,j}^{n+1} + C_1 \hat{u}_{2i,j}^{n+1} & = F_1 \\ A_2 \hat{u}_{1i,j}^{n+1} + B_2 \hat{u}_{2i,j}^{n+1} + C_2 \hat{u}_{3i,j}^{n+1} & = F_2 \\ A_3 \hat{u}_{2i,j}^{n+1} + B_3 \hat{u}_{3i,j}^{n+1} & = F_3 \end{cases} \quad (3.18)$$

其中

$$\begin{cases} C_1 = -2v_2 \Delta t / [h'_{1i,j}(h'_{1i,j} + h'_{2i,j})] \\ B_1 = 1 - C_1 \\ A_2 = C_1 h'_{1i,j} / h'_{2i,j} \\ C_2 = -2v_3 \Delta t / [h'_{2i,j}(h'_{2i,j} + h'_{3i,j})] \\ B_2 = 1 - A_2 - C_2 \\ A_3 = C_2 h'_{2i,j} / h'_{3i,j} \\ B_3 = 1 - A_3 \\ F_1 = u_{1i,j}^n + \Delta t \{ f \bar{v}_{1i,j}^n - G_1 + \tau_{sxi,j}^{n+1/2} / h'_{1i,j} + A[(u_{1i-1,j}^n + u_{1i+1,j}^n - 2u_{1i,j}^n) / \Delta x^2 \\ \quad + (u_{1i,j}^n + u_{1i,j+1}^n - 2u_{1i,j}^n) / \Delta y^2] \} \\ F_2 = u_{2i,j}^n + \Delta t \{ f \bar{v}_{2i,j}^n - G_2 + A[(u_{2i-1,j}^n + u_{2i+1,j}^n - 2u_{2i,j}^n) / \Delta x^2 \\ \quad + (u_{2i,j}^n + u_{2i,j+1}^n - 2u_{2i,j}^n) / \Delta y^2] \} \\ F_3 = u_{3i,j}^n + \Delta t \{ f \bar{v}_{3i,j}^n - G_3 - C[(u_{3i,j}^n)^2 + (\bar{v}_{3i,j}^n)^2]^{1/2} u_{3i,j}^n / h'_{3i,j} \\ \quad + A[(u_{3i-1,j}^n + u_{3i+1,j}^n - 2u_{3i,j}^n) / \Delta x^2 + (u_{3i,j-1}^n + u_{3i,j+1}^n - 2u_{3i,j}^n) / \Delta y^2] \} \end{cases} \quad (3.19)$$

其中

$$\begin{cases} G_1 = [g(\zeta_{1i,j}^{n+1/2} - \zeta_{1i-1,j}^{n+1/2}) + (\tilde{p}_{i,j}^{n+1/2} - \tilde{p}_{i-1,j}^{n+1/2})] / \Delta x \\ G_2 = (\rho_1 / \rho_2) G_1 + g(1 - \rho_1 / \rho_2)(\zeta_{2i,j}^{n+1/2} - \zeta_{2i-1,j}^{n+1/2}) / \Delta x \\ G_3 = (\rho_2 / \rho_3) G_2 + g(1 - \rho_2 / \rho_3)(\zeta_{3i,j}^{n+1/2} - \zeta_{3i-1,j}^{n+1/2}) / \Delta x \end{cases} \quad (3.20)$$

最后取

$$u_{1i,j}^{n+1} = \hat{u}_{1i,j}^{n+1} + \Delta u, \quad u_{2i,j}^{n+1} = \hat{u}_{2i,j}^{n+1} + \Delta u, \quad u_{3i,j}^{n+1} = \hat{u}_{3i,j}^{n+1} + \Delta u \quad (3.21)$$

其中

$$\Delta u = u_{i,j}^{n+1} - (\hat{u}_{1i,j}^{n+1} h'_{1i,j} + \hat{u}_{2i,j}^{n+1} h'_{2i,j} + \hat{u}_{3i,j}^{n+1} h'_{3i,j}) / h'_{i,j} \quad (3.22)$$

(b) v_k 的计算

若 $K''_{i,j} = 1$, 可直接取

$$v_{1i,j}^{n+1} = v_{i,j}^{n+1} \quad (3.23)$$

若 $K''_{i,j} = 2$, 则

$$\begin{cases} \hat{v}_{1i,j}^{n+1} = (B_2 F_1 - C_1 F_2) / (1 - A_2 - C_1) \\ \hat{v}_{2i,j}^{n+1} = (B_1 F_2 - A_2 F_1) / (1 - A_2 - C_1) \end{cases} \quad (3.24)$$

其中

$$\begin{cases} C_1 = -2v_2 \Delta t / [h''_{1i,j} (h''_{1i,j} + h''_{2i,j})] \\ B_1 = 1 - C_1 \\ A_2 = C_1 h''_{1i,j} / h''_{2i,j} \\ B_2 = 1 - A_2 \\ F_1 = v_{1i,j}^n + \Delta t \{ -f \bar{u}_{1i,j}^n - G_1 + \tau_{syn}^{n+1/2} / h''_{1i,j} + A [(v_{1i-1,j}^n + v_{1i+1,j}^n - 2v_{1i,j}^n) / \Delta x^2 \\ + (v_{1i,j-1}^n + v_{1i,j+1}^n - 2v_{1i,j}^n) / \Delta y^2] \} \\ F_2 = v_{2i,j}^n + \Delta t \{ -f \bar{u}_{2i,j}^n - G_2 - C [(\bar{u}_{2i,j}^n)^2 + (v_{2i,j}^n)^2]^{1/2} v_{2i,j}^n / h''_{2i,j} \\ + A [(v_{2i-1,j}^n + v_{2i+1,j}^n - 2v_{2i,j}^n) / \Delta x^2 + (v_{2i,j-1}^n + v_{2i,j+1}^n - 2v_{2i,j}^n) / \Delta y^2] \} \end{cases} \quad (3.25)$$

式中

$$\begin{cases} G_1 = [g(\zeta_{1i,j}^{n+1/2} - \zeta_{1i,j-1}^{n+1/2}) + (\tilde{p}_{i,j}^{n+1/2} - \tilde{p}_{i,j-1}^{n+1/2})] / \Delta y \\ G_2 = (\rho_1 / \rho_2) G_1 + g(1 - \rho_1 / \rho_2) (\zeta_{2i,j}^{n+1/2} - \zeta_{2i,j-1}^{n+1/2}) \Delta y \end{cases} \quad (3.26)$$

最后取

$$v_{1i,j}^{n+1} = \hat{v}_{1i,j}^{n+1} + \Delta v, \quad v_{2i,j}^{n+1} = \hat{v}_{2i,j}^{n+1} + \Delta v \quad (3.27)$$

其中

$$\Delta v = v_{i,j}^{n+1} - (\hat{v}_{1i,j}^{n+1} h''_{1i,j} + \hat{v}_{2i,j}^{n+1} h''_{2i,j}) / h''_{i,j} \quad (3.28)$$

若 $K''_{i,j} = 3$, 则 $\hat{v}_k^{n+1}$ 由下列方程解出

$$\begin{cases} B_1 \hat{v}_{1,j}^{n+1} + C_1 \hat{v}_{2,j}^{n+1} & = F_1 \\ A_2 \hat{v}_{1,j}^{n+1} + B_2 \hat{v}_{2,j}^{n+1} + C_2 \hat{v}_{3,j}^{n+1} & = F_2 \\ A_3 \hat{v}_{2,j}^{n+1} + B_3 \hat{v}_{3,j}^{n+1} & = F_3 \end{cases} \quad (3.29)$$

其中

$$\begin{cases} C_1 = -2v_2 \Delta t / [h''_{1i,j}(h''_{1i,j} + h''_{2i,j})] \\ B_1 = 1 - C_1 \\ A_2 = C_1 h''_{1i,j} / h''_{2i,j} \\ C_2 = -2v_3 \Delta t / [h''_{2i,j}(h''_{2i,j} + h''_{3i,j})] \\ B_2 = 1 - A_2 - C_2 \\ A_3 = C_2 h''_{2i,j} / h''_{3i,j} \\ B_3 = 1 - A_3 \\ F_1 = v_{1i,j}^n + \Delta t \{ -f \bar{u}_{1i,j}^n - G_1 + \tau_{xy,j}^{n+1/2} / h''_{1i,j} + A[(v_{1i-1,j}^n + v_{1i+1,j}^n - 2v_{1i,j}^n) / \Delta x^2 \\ + (v_{1i,j-1}^n + v_{1i,j+1}^n - 2v_{1i,j}^n) / \Delta y^2] \} \\ F_2 = v_{2i,j}^n + \Delta t \{ -f \bar{u}_{2i,j}^n - G_2 + A[(v_{2i-1,j}^n + v_{2i+1,j}^n - 2v_{2i,j}^n) / \Delta x^2 \\ + (v_{2i,j-1}^n + v_{2i,j+1}^n - 2v_{2i,j}^n) / \Delta y^2] \} \\ F_3 = v_{3i,j}^n + \Delta t \{ -f \bar{u}_{3i,j}^n - G_3 - C[(\bar{u}_{3i,j}^n)^2 + (v_{3i,j}^n)^2]^{1/2} v_{3i,j}^n / h''_{3i,j} \\ + A[(v_{3i-1,j}^n + v_{3i+1,j}^n - 2v_{3i,j}^n) / \Delta x^2 + (v_{3i,j-1}^n + v_{3i,j+1}^n - 2v_{3i,j}^n) / \Delta y^2] \} \end{cases} \quad (3.30)$$

式中

$$\begin{cases} G_1 = [g(\zeta_{1i,j}^{n+1/2} - \zeta_{1i,j-1}^{n+1/2}) + (\tilde{p}_{i,j}^{n+1/2} - \tilde{p}_{i,j-1}^{n+1/2})] / \Delta y \\ G_2 = (\rho_1 / \rho_2) G_1 + g(1 - \rho_1 / \rho_2) (\zeta_{2i,j}^{n+1/2} - \zeta_{2i,j-1}^{n+1/2}) / \Delta y \\ G_3 = (\rho_2 / \rho_3) G_2 + g(1 - \rho_2 / \rho_3) (\zeta_{3i,j}^{n+1/2} - \zeta_{3i,j-1}^{n+1/2}) / \Delta y \end{cases} \quad (3.31)$$

解出 $\hat{v}_k$ 后, v_k 值由 $\hat{v}_k$ 加上一个订正得到:

$$v_{1i,j}^{n+1} = \hat{v}_{1i,j}^{n+1} + \Delta v, \quad v_{2i,j}^{n+1} = \hat{v}_{2i,j}^{n+1} + \Delta v, \quad v_{3i,j}^{n+1} = \hat{v}_{3i,j}^{n+1} + \Delta v \quad (3.32)$$

其中

$$\Delta v = v_{i,j}^{n+1} - (\hat{v}_{1i,j}^{n+1} h''_{1i,j} + \hat{v}_{2i,j}^{n+1} h''_{2i,j} + \hat{v}_{3i,j}^{n+1} h''_{3i,j}) / h''_{i,j} \quad (3.33)$$

从式(3.30)可知, $u_1(i, j), u_2(i, j), u_3(i, j), v_1(i, j), v_2(i, j), v_3(i, j)$ 在计算过程中需使用两个数组。

(2) ζ_k 的计算

ζ_2 和 ζ_3 可用简单的显式计算:

$$\begin{cases} \zeta_{3i,j}^{n+3/2} = \zeta_{3i,j}^{n+1/2} - \Delta t \{ (h'_{3i+1,j} u_{3i+1,j}^{n+1} - h'_{3i,j} u_{3i,j}^{n+1}) / \Delta x \\ \quad + (h''_{3i,j+1} v_{3i,j+1}^{n+1} - h''_{3i,j} v_{3i,j}^{n+1}) / \Delta y \} \\ \zeta_{2i,j}^{n+3/2} = \zeta_{2i,j}^{n+1/2} + \zeta_{3i,j}^{n+3/2} - \zeta_{3i,j}^{n+1/2} - \Delta t \{ (h'_{2i+1,j} u_{2i+1,j}^{n+1} - h'_{2i,j} u_{2i,j}^{n+1}) / \Delta x \\ \quad + (h''_{2i,j+1} v_{2i,j+1}^{n+1} - h''_{2i,j} v_{2i,j}^{n+1}) / \Delta y \} \end{cases} \quad (3.34)$$

由上式知 $\zeta_k (k=2,3)$ 只要一个数组,其中第二式用到 $\zeta_{3i,j}^{n+3/2}$,可用一个单元暂存。

四、初步应用结果

作为对模式可用性的一次检验,我们选取了南海西北部潮波进行试算。计算区域包括 15°N 以北, 115°E 以西的海域,网格距离大约 $1/8$ 度,即取 $\Delta x = 13.132\text{km}$, $\Delta y = 13.889\text{km}$, $\Delta t = 89.4283\text{s}$ 。其他参数为: $h_1 = 200\text{m}$, $h_2 = 200\text{m}$, $\rho_1 = 1021\text{kg/m}^3$, $\rho_2 = 1024\text{kg/m}^3$, $\rho_3 = 1027\text{kg/m}^3$, $A = 100\text{m}^2/\text{s}$, $v_2 = v_3 = 0.05\text{m}^2/\text{s}$, $C = 0.0025$ 。由于只模拟潮波,海面风应力忽略,因而开边界以 m_1 和 M_2 分潮潮汐高度输入, m_1 的定义为 K_1 和 O_1 的平均值。

模拟得到的 m_1 和 M_2 见图3和图4,将这些图与由观测得到的同潮图(Fang, 1986)相

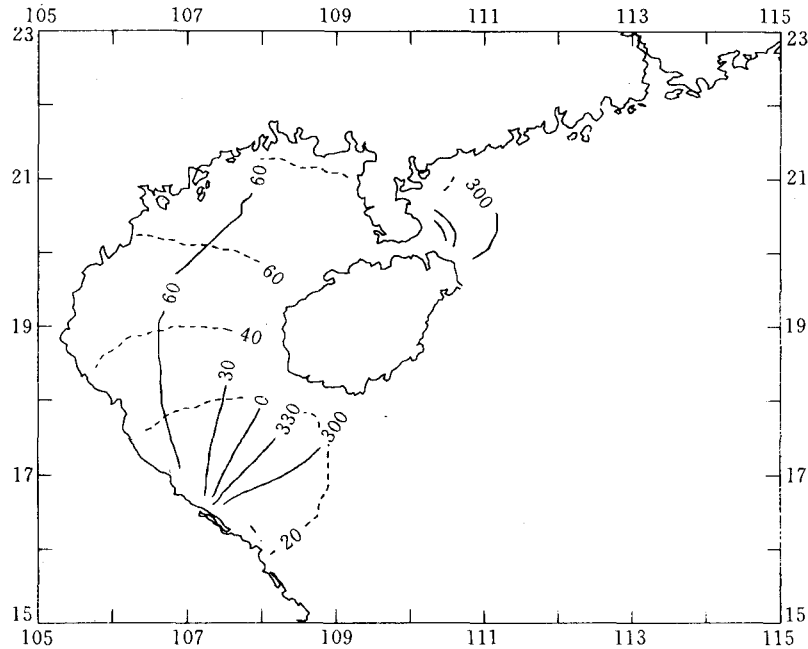
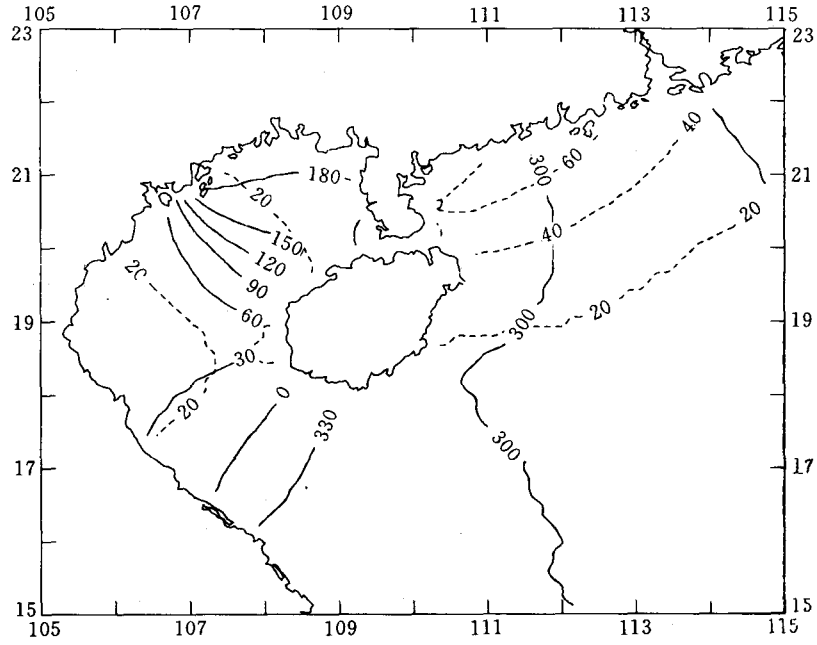
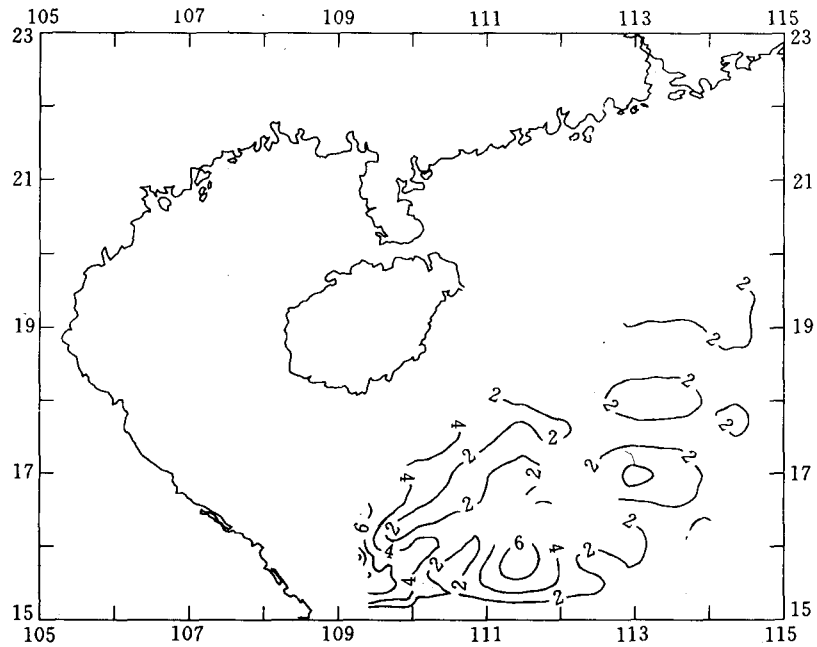


图3 模拟得到的 m_1 表面同潮图

----振幅(单位为 cm); ——迟角(单位为度)

图4 模拟得到的 M_2 表面同潮图

--- 振幅(单位为 cm); — 迟角(单位为度)

图5 第一与第二界面处的 m_1 内潮振幅分布(m)

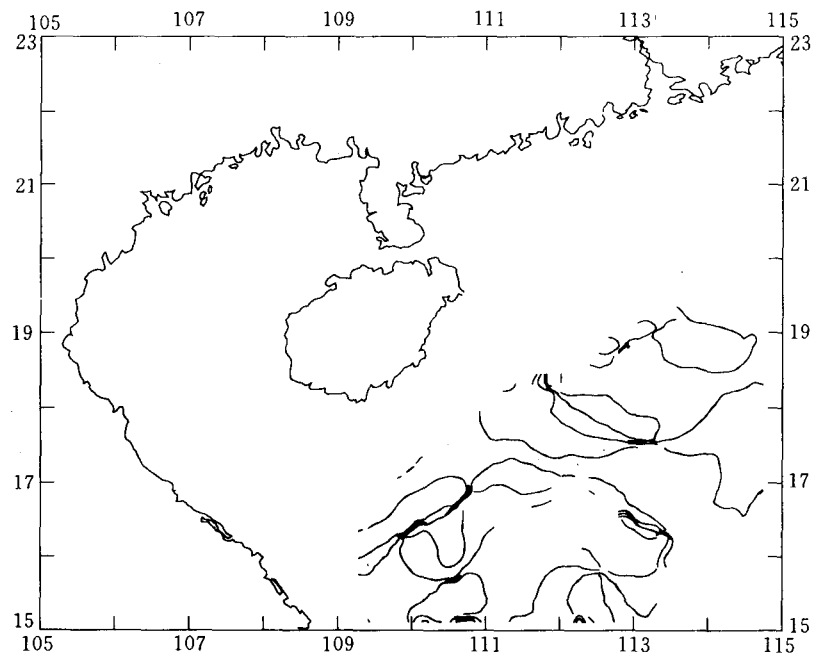


图6 第一与第二界面处的 m_1 内潮迟角分布

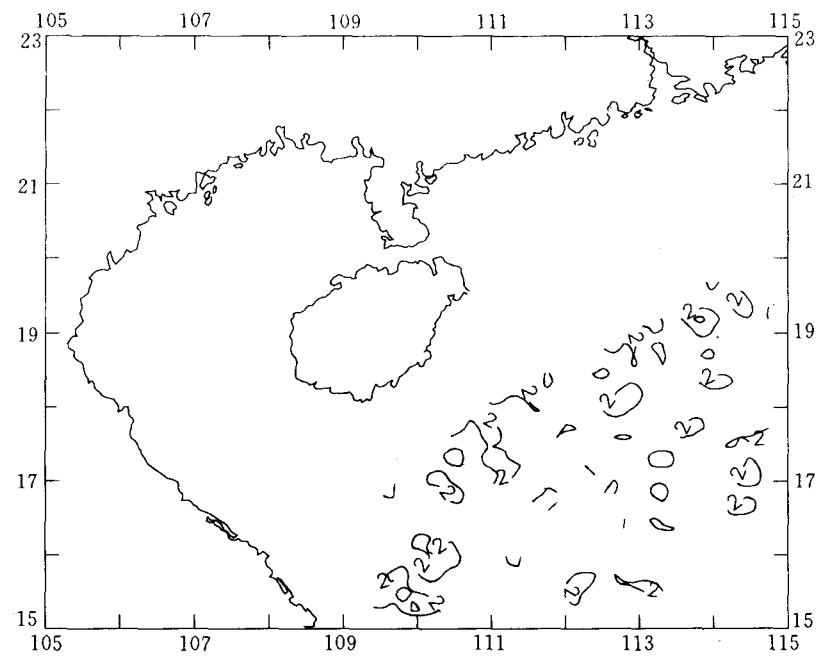


图7 第一与第二界面处的 M_2 内潮振幅分布(m)

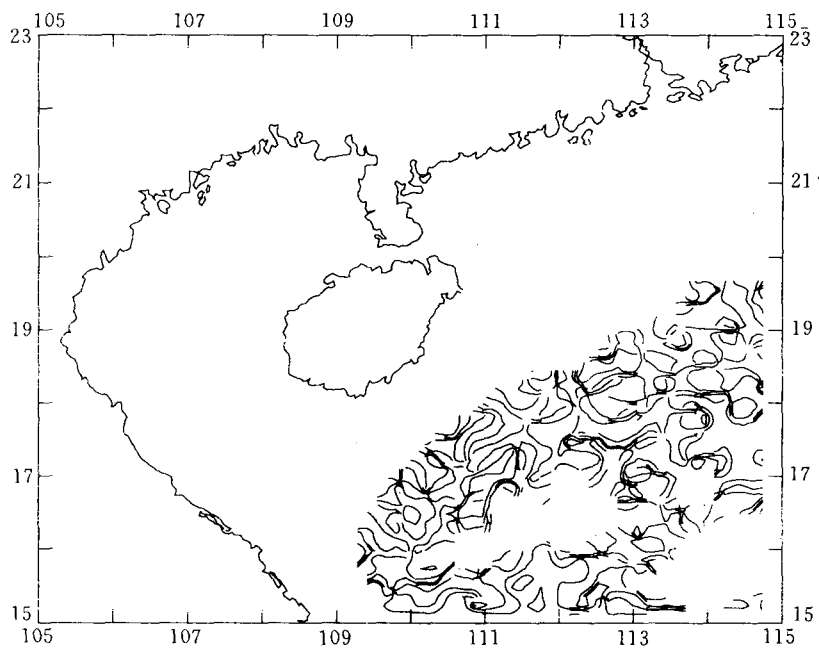


图8 第一与第二界面处的 M_2 内潮迟角分布

比较,可见本模式能良好地模拟表面潮波。

图5—8示出第一层和第二层之间界面上的 m_1 和 M_2 ,由这些图可以看出如下两个特征:

1. 内潮的振幅比表面潮要大得多,在本海区全日内潮更强。例如 M_2 可过 2m, m_1 则可超过 6m。
2. 内潮具有比表面潮短得多的波长,且半日潮波长比全日潮更短,因而同潮时线的分布具有很复杂的结构。

限于篇幅,这里不给出第二层和第三层之间界面的同潮图,它们的具体分布情况与上一界面的分布有很大不同,但基本特征与上述一致。

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A LAYERED 3-D NUMERICAL OCEAN MODEL FOR SIMULATION OF INTERNAL TIDES*

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ABSTRACT

In this layered three-dimensional numerical ocean model the seasurface is free to allow simultaneous simulations of surface and internal tides. Since the propagating speed of the external wave is much greater than those of internal waves, the Courant-Friedrichs-Lewy (CFL) condition for external mode is much more restrictive than that for internal modes. In the present model an alternating direction implicit (ADI) scheme is employed for external mode computation to avoid the restriction of CFL condition. The internal modes are computed with an explicit treatment.

The model has been preliminarily applied for simulation of internal tides generated by surface tides in the northwestern South China Sea.

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